S–7512

Sub. Code
22MMA1C1

M.Sc. DEGREE EXAMINATION, APRIL 2025

First Semester

Mathematics

ALGEBRA – I

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define a normal subgroup with an example.

2. Express the product of disjoint cycles :

   $(1,\ 2,\ 3)(4,\ 5)\ (1,\ 6,\ 7,\ 8,\ 9)\ (1,\ 5)$

3. Prove that $N(a)$ is a subgroup of G.

4. Show that any group of order 15 is cyclic?

5. State the pigeonhole principle.

6. If p is a prime number, then prove that J_p (the ring of integers mod p) is a field.

7. If F is a field, then prove that its only ideals are $\{0\}$ and F itself.

8. Define maximal ideal of R.

9. Define the division algorithm.

10. Prove that a Euclidean ring possesses a unit element.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Prove that if G is a finite group and N is a normal subgroup of G, then prove that $O(G/N) = O(G)/O(N)$.

Or

(b) If ϕ is a homomorphism of G into $\overline{G}$ with Kernel K, then prove that K is a normal subgroup of G.

12. (a) Suppose that G is the internal direct product of $N_1, N_2, ..., N_n$, then prove that for $i \neq j$, $N_i \cap N_j = \{e\}$ and if $a \in N_i$, $b \in N_j$ then prove that $ab = ba$.

Or

(b) State and prove Cauchy's theorem.

13. (a) Prove that any field is an integral domain.

Or

(b) Show that the commutative ring D is an integral domain if and only if for $a, b, c \in D$ with $a \neq 0$, the relation $ab = bc$ implies that $b = c$.

14. (a) If R is a commutative ring with unit element and M is an ideal of R, then prove that M is an Maximal ideal of R if and only if R/M is a field.

Or

(b) Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Then prove that R is a field.

15. (a) State and prove Fermat's theorem.

Or

(b) Let R be a Euclidean ring and $a, b \in R$. If $b \neq 0$ is not a unit in R then prove that $d(a) < d(ab)$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. State and prove Sylow's theorem for Abelian group.

17. Let G be a group and suppose of G is the internal direct product of $N_1, N_2, ..., N_n$. Let $T = N_1 \times N_2 \times ... \times N_n$ then prove that G and T are isomorphic.

18. Using the pigeonhole principle, prove that if m and n are relatively prime integers and a and b are any integers, there exists an integer x such that $x \equiv a \bmod m$ and $x \equiv b \bmod n$.

19. Let R be a ring, possibly non commutative in which $xy = 0$ implies $x = 0$ (or) $y = 0$. If $a, b \in R$ and $a^n = b^n$ and $a^m = b^m$ for two relatively prime positive integers m and n, then prove that $a = b$.

20. State and prove the Eisenstein criterion theorem.

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S–7514

Sub. Code
22MMA2C4

M.Sc. DEGREE EXAMINATION, APRIL 2025

Second Semester

Mathematics

PROBABILITY AND STATISTICS

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Find the constant C so that $f(x) = C\left(\frac{2}{3}\right)^x, x = 1, 2, ...,$ zero, elsewhere. satisfies the condition of being a p.d.f. of one random variable X.

2. Let the space of the random variable x be $A = \{x; 0 < x < 1\}$ of $A_1 = \left\{x; 0 < x < \frac{1}{2}\right\}$ and $A_2 = \left\{x; \frac{1}{2} x < 1\right\}$. Find $P(A_2)$ is $P(A_1) = \frac{1}{4}$.

3. Let x_1 and x_2 have the joint p.d.f. $f(x_1, x_2) = x_1 + x_2$, $0 < x_1 < 1, 0 < x_2 < 1$, zero elsewhere. Find each marginal p.d.f.

4. Let x_1 and x_2 have the joint p.d.f. $f(x_1, x_2) = 12x_1x_2(1 - x_2)$, $0 < x_1 < 1, 0 < x_2 < 1$, zero elsewhere. Show that the random variables x_1 and x_2 are independent.

5. Let X be the number of needs (successes) in $n=7$ independent tosses of an unbiased coin: The p.d.f. of X is $f(x)=\binom{7}{x}\left(\frac{1}{2}\right)^x\left(1-\frac{1}{2}\right)^{7-x}, x=0,1,2,....7,$ zero elsewhere. Find $\Pr(X=5)$.

6. The m.g.f. of a random variable x is $e^{4\left(e^t-1\right)}$. Show that $pr(\mu-2\sigma<x<\mu+2\sigma)=0.931$.

7. Let $\overline{X}$ be the mean of a random sample of size n from a normal distribution with mean μ and variance 100. Find n so that $\Pr(\mu-5<\overline{x}<\mu+5)=0.954$.

8. Define a statistic.

9. Let X be $\chi^2(50)$. Approximate $\Pr(40<x<60)$.

10. Let z_n be $\chi^2(n)$. Find the mean and variance of z_n.

Part B (5 × 5 = 25)

Answer **all** questions, choosinpg either (a) or (b).

11. (a) Let x be a random variance of the discrete type with p.d.f $f(x)=\frac{x}{6}$, $x=1,2,3,$ zero elsewhere. Find the distribution function and the p.d.f. of $Y=x^2$.

Or

(b) Let x have the p.d.f. $f(x)=\frac{x+2}{18}, -2<x<4,$ zero elsewhere. Find $E(x), E\left[(x+2)^2\right],$ and $E\left[6x-2(x+2)^3\right]$.

12. (a) Let $f(x_1, x_2) = 21 x_1^2 x_2^2, 0 < x_1 < x_2 < 1$, zero elsewhere. Be the joint p.d.f of x_1 and x_2.

(i) Find the conditional mean and variance of X_1 given $X_2 = x_2,\ 0 < x_2 < 1$.

(ii) Find the distribution of $Y = E(x_1 | x_2)$. Also determine $E(Y)$ and $\text{var}(Y)$.

Or

(b) If x_1 and x_2 are independent random variable with marginal probability density functions $f_1(x_1)$ and $f_2(x_2)$, respectively, then show that $\Pr(a < x_1 < b, c < x_2 < d)$ for every $a < b$ and $c < d$, where a, b, c and d are constants.

13. (a) Compute the measures of skewness and Kurtosis of a gamma distribution with parameters α and β.

Or

(b) If the random variable x is $N(\mu, \sigma^2), \sigma^2 > 0$, then show that the random variable $v = \dfrac{(x-\mu)^2}{\sigma^2}$ is $\chi^2(1)$.

14. (a) Let x_1 and x_2 have the joint p.d.f. $f(x_1, x_2) = \dfrac{x_1 x_2}{36}$, $x_1 = 1, 2, 3$ and $x_2 = 1, 2, 3$, zero elsewhere. Find first the joint p.d.f. of $Y_1 = X_1 X_2$ and $Y_2 = X_2$ and then find the marginal p.d.f. of Y_1.

Or

(b) Derive the double exponential p.d.f.

15. (a) Let $\overline{X}_n$ denote the mean of a random sample of size n from a distribution that is $N(\mu, \sigma^2)$. Find the limiting distribution of $\overline{x}_n$.

Or

(b) Let z_n be $\chi^2(n)$ and let $w_n = \frac{z_n}{n^2}$. Find the limiting distribution of w_n.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) Find the mean and variance for the distribution $f(x) = \frac{3!}{x!(3-x)!}\left(\frac{1}{2}\right)^3, x = 0,1,2,3,$ zero elsewhere.

(b) State and prove the Chebyshev's inequality.

17. Let $f(x) = 2,\ 0 < x < y, 0 < y < 1$, zero elsewhere be the joint p.d.f. of x and y. Show that the conditional means are respectively $\left(\frac{1+x}{2}\right), 0 < x < 1$ and $\frac{y}{2}, 0 < y < 1$. Show that the correlation coefficient of x and y is $\rho = \frac{1}{2}$.

18. Find the moment generating function, mean and variance of the normal distribution.

19. Derive the p.d.f. of T-distribution.

20. State and prove the central limit theorem.

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S–7515

Sub. Code
22MMA2E2

M.Sc. DEGREE EXAMINATION, APRIL 2025

Second Semester

Mathematics

Elective — NUMERICAL METHODS

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. How the constant should α be chosen to ensure the fastest possible convergence with the iteration formula $x_{n+1} = \dfrac{\alpha x_n + x_n^{-2} + 1}{\alpha + 1}$.

2. Prove that Newton Raphson method has second order convergence.

3. What is vector norm? Give the properties.

4. Define :

   (a) Relaxation parameter

   (b) Over relaxation and

   (c) Under relaxation method.

5. List the disadvantages of Quadratic splines.

6. What is a cubic spline?

7. Define the order of a numerical differentiation method.

8. What is meant by Extrapolation method?

9. What is the order of error in Simpson's formula?

10. What is the order of error in trapezoidal formula?

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Find the number of real and complex roots of the polynomial equation $P_4(x) = x^4 - 4x^3 + 3x^2 + 4x - 4 = 0$ using Sturm sequences.

Or

(b) Perform two iterations of the Bairstow method to extract a quadratic factor $x^2 + px + q$ from the polynomial $P_3(x) = x^3 + x^2 - x + 2 = 0$. Use the initial approximation $p_0 = -0.9, q_0 = 0.9$.

12. (a) Find the largest eigenvalue on modulus and the corresponding eigenvector of the matrix $A = \begin{bmatrix} -15 & 4 & 3 \\ 10 & -12 & 6 \\ 20 & -4 & 2 \end{bmatrix}$ using the power method.

Or

(b) For the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$

(i) Find all the eigen values and the corresponding eigenvectors.

(ii) Verify $S^{-1}AS$ is a diagonal matrix, where S is the matrix of eigenvectors.

13. (a) Using the following values of $f(x)$ and $f'(x)$.

x	$f(x)$	$f'(x)$
–1	1	–5
0	1	1
1	3	7

Estimate the values of $f(-0.5)$ and $f(0.5)$ using piecewise cubic Hermite interpolation.

Or

(b) Given the data

x	0	1	2	3
$f(x)$	1	2	33	244

Fit quadratic splines with $M(0) = f''(0) = 0$. Hence, find an estimate of $f(0.5)$.

14. (a) A differentiation rule of the form $hf'(x_2) = \alpha_0 f(x_0) + \alpha_1 f(x_1) + \alpha_2 f(x_3) + \alpha_3 f(x_4)$ where $x_j = x_0 + jh,\ j = 0, 1, 2, 3, 4$ is given. Determine the values of $\alpha_0, \alpha_1, \alpha_2$ and α_3 so that the rule is exact for a polynomial of degree 4. Also find an expression for the round-off error in calculating $f'(x_2)$.

Or

(b) Define $S(h) = \dfrac{-y(x+2h) + 4y(x+h) - 3y(x)}{2h}$ show that $y'(x) - S(h) = c_1 h^2 + c_2 h^3 + c_3 h^4 ....$ and state c_1.

15. (a) Find the approximate value of $I = \int_0^1 \dfrac{dx}{1+x}$, using Simpson's three – eight rule.

Or

(b) Evaluate the integral $I = \int_0^1 \dfrac{dx}{1+x}$, using Gauss – Legendre three point formula.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Obtain the complex roots of the equation $f(z) = z^3 + 1 = 0$ correct to eight decimal places. Use the initial approximation to a root as $(x_0, y_0) = (0.25, 0.25)$. Compare with the exact values of the roots $\dfrac{\left(1 + i\sqrt{3}\right)}{2}$.

17. Consider the system of equations

$$\begin{aligned} 2x - y &= 1 \\ -x + 2y - z &= 0 \\ -y + 2z - w &= 0 \\ -z + 2w &= 1 \end{aligned}$$

(a) Set up the Gauss – Seidel iteration scheme in matrix form. Show that the scheme converges and hence find its rate of convergence.

(b) Starting with $x^{(0)} = 0$ as initial approximation, iterate three times.

18. Obtain the cubic spline approximation for the function defined by the data

x	0	1	2	3
$f(x)$	1	2	33	244

with $M(0) = 0, M(3) = 0$. Hence, find an estimate of $f(2.5)$.

19. Derive the formulas for the first derivative of $y = f(x)$ of $O(h^2)$ using

(a) Forward difference approximations,

(b) Backward difference approximations,

(c) Central difference approximations.

20. Find the quadrature formula. $\int_0^1 f(x)\frac{dx}{\sqrt{x(1-x)}} = \alpha_1 f(0) + \alpha_2 f\left(\frac{1}{2}\right) + \alpha_3 f(1)$, which is exact for polynomials of highest possible degree. Then use the formula on $\int_0^1 \frac{dx}{\sqrt{x - x^3}}$ and compare with the exact value.

S–7516

Sub. Code
22MMA3C1

M.Sc. DEGREE EXAMINATION, APRIL 2025

Third Semester

Mathematics

COMPLEX ANALYSIS

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define a power series. Give an example.

2. Distinguish between translation, rotation and inversion.

3. Compute $\int_{\gamma} x dz$ where γ is the directed line segment from 0 to $1+i$.

4. State the Liouville's theorem.

5. Show that the functions $\cos z$ have essential singularity at ∞.

6. State the maximum principle theorem.

7. Find the residue of $\cot z$ at $z=0$.

8. State the argument principle theorem.

9. Write down the formula for the series expansion of arc $\sin z$.

10. State the Poisson-Jensen formula.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) State and prove complex form of the Cauchy-Riemann equations.

Or

(b) If Z_1, Z_2, Z_3, Z_4 are distinct points in the extended plane and T any linear transformation, then prove that $(T_{Z_1}, T_{Z_2}, T_{Z_3}, T_{Z_4}) = (Z_1, Z_2, Z_3\, Z_4)$.

12. (a) Prove that the line integral $\int_\gamma pdx + qdy$, defined in Ω, depends only on the end points of γ if and only if there exists a function $U(x, y)$ in Ω with the partial derivatives $\partial u/\partial x = p$ and $\partial u/\partial y = q$.

Or

(b) State and prove the Morera's theorem.

13. (a) State and prove the Wierestrass theorem for essential singularity.

Or

(b) State and prove the local mapping theorem.

14. (a) State and prove the residue theorem.

Or

(b) Evaluate $\int_0^\infty \frac{\cos x}{x^2 + a^2} dx$, a real.

15. (a) With the usual notations, prove that $\frac{\pi^2}{\sin^2 \pi^2} = \sum_{-\infty}^{\infty} \frac{1}{(z-n)^2}$.

Or

(b) (i) Prove that every function which is mesomorphic in the whole plane is the quotient of two entire functions.

(ii) What is the genus of $\cos\sqrt{z}$?

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) State and prove the Abel's limit theorem.

(b) If $T_1(Z) = \frac{Z+2}{Z+3}$, $T_2(Z) = \frac{Z}{Z+1}$, find $T_1T_2(Z)$, $T_2T_1(Z)$ and $T_1^{-1}T_2(Z)$.

17. State and prove the Cauchy's integral formula. Also deduce that $f_{(z)}^{(n)} = \frac{n!}{2\pi i}\int \frac{f(r)}{(r-z)^{n+1}}dr$.

18. State and prove the Schwarz lemma.

19. Show that $\int_0^{\pi} \log \sin x dx = -\pi \log 2$.

20. Obtain the Laurent expansion $\sum_{n=-\infty}^{\infty} A_n (z-a)^n$ for the function $f(z)$ analytic in $R_1 < |z-a| < R_2$.

S–7517

Sub. Code
22MMA3C2

M.Sc. DEGREE EXAMINATION, APRIL 2025

Third Semester

Mathematics

TOPOLOGY – I

(CBCS – 2022 onwards)

Time : 3 Hours — Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define the discrete topology. Give an example.

2. Is the subset $[a,b]$ of $\mathbb{R}$ closed? Justify your answer.

3. Can you conjecture what functions $f:\mathbb{R}\to\mathbb{R}$ are continuous when considered as maps from $\mathbb{R}$ to $\mathbb{R}_l$?

4. In $\mathbb{R}^n$, define $d'(x,y)=|x_1-y_1|+|x_2-y_2|+\cdots+|x_n-y_n|$. Show that d' is is a metric that induces the usual topology of $\mathbb{R}^n$.

5. Is the rationals Q connected? Justify your answer.

6. What is meant by punctured Euclidean space?

7. State the tube lemma.

8. When will you say that a space X is said to be limit point compact?

9. Prove that a subspace of a Lindelof space need not be Lindelof.

10. Check whether the space $\mathbb{R}_k$ is Hansdorff or not? Justify your answer.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Define the subspace topology with an example. Also if $\mathfrak{B}$ is a basis for the topology of x, then prove that the collection $\mathfrak{B}_y = \{B \cap Y / B \in \mathfrak{B}\}$ is a basis for the subspace topology on Y.

Or

(b) Let X be a topological space. Prove the following conditions hold:

(i) ϕ and X are closed.

(ii) Arbitrary intersections of closed sets are closed

12. (a) State and prove the pasting lemma.

Or

(b) Let $f : A \to \prod_{\alpha \in J} X_\alpha$ be given by the equation. $f(a) + (f_\alpha(a))_{\alpha \in J}$, where $f_\alpha : A \to X_\alpha$ for each α. Let $\prod X_\alpha$ have the product topology. Prove that the function f is continuous if and only if each function f_α is continuous.

13. (a) Prove that the image of a connected space under a continuous map is connected.

Or

(b) Show that a space X is locally connected if and only if for every open set if U of X, each component of U is open in X.

14. (a) State and prove the uniform continuity theorem.

Or

(b) Prove that compactness implies limit point compactness, but not conversely.

15. (a) Define the following terms. Give an example for each.

(i) Dense set; (ii) G_δ – set; (iii) Regular space; (iv) Normal space.

Or

(b) Show that a subspace of a completely regular space is completely regular.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) Let $\mathfrak{B}$ and $\mathfrak{B}'$ be bases for the topologies $\mathcal{J}$ and $\mathcal{J}'$, respectively, on X. Prove the following are equivalent:

(i) $\mathcal{J}'$ is finer that $\mathcal{J}$.

(ii) For each $x \in X$ and each basis element $B \in \mathfrak{B}$ containing x, there is a basis element $B' \in \mathfrak{B}'$ such that $x \in B' \subset B$.

(b) Show that the topologies of $\mathbb{R}_l$ and $\mathbb{R}_k$ are not comparable.

17. Prove that the topologies on $\mathbb{R}^n$ induced by the Euclidean metric d and the square metric p are the same as the product topology on $\mathbb{R}^n$.

18. If L is a linear continuum in the order topology, then prove that L is connected, and so are intervals and rays in L.

19. (a) Show that every compact subspace of a Hausdorff space is closed.

(b) Let $f : x \to y$ be a bijective continuous function. If X is compact and Y is Hausdorff, then prove that f is homeomorphism.

20. State and prove the Urysohn metrization theorem.

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S–7518

Sub. Code
22MMA3C3

M.Sc. DEGREE EXAMINATION, APRIL 2025

Third Semester

Mathematics

GRAPH THEORY

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define a regular graph with an example.

2. What is meant by a forest? Give an example.

3. Define a cut vertex. Give an example.

4. Draw a graph which is Eulerian but not Hamiltonian.

5. Find the number of different perfect matchings in K_{2n}.

6. Define the edge chromatic number of a graph. Give an example.

7. Define an independent set of a graph with an example.

8. What is meant by critical graph?

9. Embed $K_{3,3}$ on mobius band.

10. Define a bridge of a graph G. Give an example.

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) Prove that in any graph, the number of vertices of odd degree is even. Also prove that $\delta \leq 2\varepsilon / \gamma \leq \Delta$, using usual notations.

Or

(b) Prove that every nontrivial tree has at least two vertices of degree one. Also prove that every tree with exactly two vertices of degree one is a path.

12. (a) Prove that a connected graph has an Euler trail if and only if it has atmost two vertices of odd degree.

Or

(b) If G is a simple graph with $\gamma \geq 3$ and $\delta \geq \gamma/2$, then prove that G is Hamiltonian.

13. (a) State and prove the Hall's theorem.

Or

(b) If G is bipartite, then prove that $\chi' = \Delta$. Also if G is bipartite, then prove that G has a Δ-regular bipartite supergraph.

14. (a) With the usual notations. If $\delta \geq 0$, then prove that $\alpha' + \beta' = \gamma$.

Or

(b) Prove that every critical graph is a block.

15. (a) (i) With the usual notations, prove that $K_{3,3}$ is non planar.

(ii) If G is a plane graph, then prove that $\sum_{f \in F} d(f) = 2\varepsilon$.

Or

(b) State the five colour theorem. Also explain the four colour conjecture.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) Let G be simple. Show that $\varepsilon = \binom{\gamma}{2}$ if and only if G is complete.

(b) Define the incidence and adjacency matrices. Give an example for each.

(c) Let M be the incidence matrix and A the adjacency matrix of a graph G. Show that every column sum of M is 2.

17. With the usual notations, prove that $k \le k' \le \delta$.

18. State and prove the Vizing's theorem.

19. If G is a connected simple graph and is neither an odd cycle nor a complete graph, then prove that $\chi \le \Delta$.

20. State and prove the Euler's formula for a connected plane graph. If G is a simple planar graph with $\gamma \le 3$, then prove that $\varepsilon \le 3\gamma - 6$.

S–7519

Sub. Code
22MMA3E3

M.Sc. DEGREE EXAMINATION, APRIL 2025

Mathematics

***Elective* – AUTOMATA THEORY**

(CBCS – 2022 onwards)

Time : 3 Hours   Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Draw a block diagram of a finite automation.

2. What is meant by "a string accepted by a finite automation"?

3. Define a sentential form.

4. What is meant by a type 3 production?

5. Is a recursive set recursively enumerable? Justify your answer.

6. Construct a context – free grammar generating $L = \{a^m b^n \mid m > n, m, n \geq 1\}$.

7. Define a regular set with an example.

8. State the Kleene's theorem.

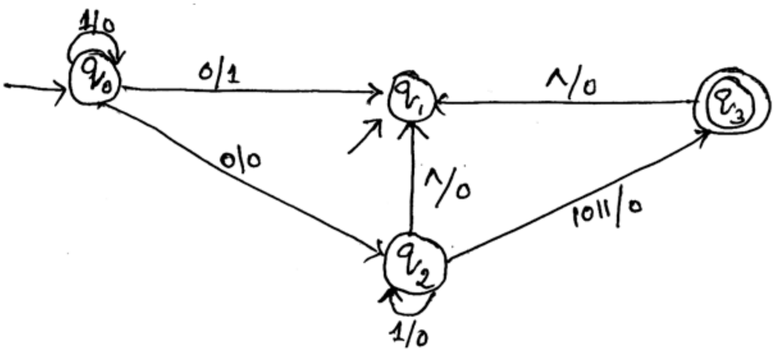
9. Define a parse tree.

10. Write short notes on Greibach normal form.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) Consider the transition system given in following figure :



Determine the initial states, the final states, and the acceptability of 101011, 111010.

Or

(b) Find a deterministic acceptor equivalent to $M = (\{q_0, q_1, q_2\}, \{a, b\}, \delta, q_0, \{q_2\})$ where δ is as given in the following table :

State/ Σ	a	b
$\rightarrow q_0$	q_0, q_1	q_2
q_1	q_0	q_1
(q_2)		q_0, q_1

12. (a) If $G = (\{S\}, \{0, 1\}, \{S \to 0S1, S \to \Lambda\}, S)$, find $L(G)$.

Or

(b) Find the highest type number which can be applied to the following productions :

(i) $S \to Aa,\ A \to C \mid Ba,\ B \to abc$

(ii) $S \to AsB \mid d,\ A \to aA$

(iii) $S \to aS \mid ab$.

13. (a) Prove that there exists a recursive set which is not a contex – sensitive language over $\{0, 1\}$.

Or

(b) Show that $\mathcal{L}_0$ is closed under transpose operation.

14. (a) State and prove the pumping lemma for regular sets.

Or

(b) If L is regular then prove that L^T is also regular.

15. (a) Consider G whose productions are $S \to aAS \mid a$, $A \to SbA \mid SS \mid ba$. Show that $S \overset{*}{\Rightarrow} aabbaa$ and construct a derivation tree whose yield is $aabbaa$.

Or

(b) Consider the grammar G whose productions are $S \to aS \mid AB$, $A \to \Lambda$, $B \to \Lambda$, $D \to b$. Construct a grammar G_1 without null productions generating $L(G) = \{\Lambda\}$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Narrate the characteristics of automation. Also prove that for any transition function δ and for any two input strings x and y, $\delta(q, xy) = \delta(\delta(q, x), y)$.

17. Let $G = (\{S, A\}, \{0, 1, 2\}, P, S)$, where P consists of $S \rightarrow 0SA$, $S \rightarrow 012$, $2A_1 \rightarrow A_1 2$, $1A_1 \rightarrow 11$. Show that $L(G) = \{0^n 1^n 2^n \mid n \geq 1\}$.

18. Prove that a context-sensitive language is recursive.

19. State and prove the Arden's theorem.

20. State and prove reduction to Chomsky normal form theorem.

S–7520

Sub. Code
22MMA4C1

M.Sc. DEGREE EXAMINATION, APRIL 2025

Fourth Semester

Mathematics

FUNCTIONAL ANALYSIS

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A ($10 \times 2 = 20$)

Answer **all** the questions.

1. State Riesz representation theorem.

2. Define operator norm.

3. What is meant by support functional normed space?

4. State Taylor–Foyuel theorem.

5. Define Banach limit.

6. What is meant by summable series?

7. What is meant by open map?

8. What is meant by approximate solution function?

9. State Polarization Identity.

10. Define orthonormal set.

Part B (5 × 5 = 25)

Answer **all** the questions choosing either (a) or (b).

11\. (a) Let X be a normed space. Let E be a convex subset of X. Then prove that the interior E° of E and the closure $\overline{E}$ of E are also convex. If $E^{\circ} \neq \phi$ then prove that $\overline{E} = \overline{E^{\circ}}$.

Or

(b) Let X be a normed space, Y be a closed subspace of X and $Y \neq X$ Let r be a real number such that $0 < r < 1$. Prove that there exists some $x_r \in X$ such that $\|x_r\| = 1$ and $r < dist(x_r, Y) \leq 1$.

12\. (a) Let X be a linear space over K and Y be a subspace of X which is not a hyperspace in X. If x_1 and x_2 are in X but in Y. Prove that there is some x in X such that for all $t \in [0,1]$. $tx_1 + (1-t)x \notin Y$ and $tx_2 + (1-t)x \notin Y$.

Or

(b) State and prove Hahn-Banach extension theorem.

13\. (a) Let X and Y be normed spaces and $X \neq \{0\}$. Prove that $BL(X,Y)$ is a Banach space in the operator norm if and only if Y is a Banach space.

Or

(b) Let x be a Banach space and (F_n) be a sequence in $BL(X,Y)$. Prove that $(Fn(x))$ converges to $F(x)$ uniformly for $x \in E$.

14. (a) Let X and Y be normed space. If Z is a closed subspace of X, then prove that the quotient map Q from X to X/Z is continuous and open.

Or

(b) Let X be a linear space over K. Consider subsets U and V of X, and $k \in K$. Such that $U \subset V + kU$. Prove that for every $x \in U$, there is a sequence $(\sqrt{n})$ in V such that $x - \left(\sqrt{1} + k\sqrt{2+} ... + k^{n-1}\sqrt{n}\right) \in k^n U, n = 1,2...$

15. (a) State and prove Parallelogram law.

Or

(b) State and prove Riesz - Fischer theorem.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Let X and Y be normed spaces and $F: X \to Y$ be a linear map. Prove that the following are equivalent :

(a) F is bounded on $\overline{U}(0, r)$ for some $r > 0$

(b) F is continuous at O

(c) F is continuous on X

(d) F is uniformly continuous on X

(e) $\|F(x)\| \leq \alpha \|x\|$ for all $x \in X$ and some $\alpha > 0$

(f) The zero space $Z(F)$ of F is closed in X and the linear map $\widetilde{F}: X/Z(F) \to Y$ defined by $\widetilde{F}:(X + Z(F) = F(X), x \in X$ is continuous

17. Let X be a normed space over K, E be a nonempty open convex subset of X and Y be a subspace of X. Such that $E \cap Y = \phi$. Prove that there is a closed hyperspace Z in X such that $Y \subset Z$ and $E \cap Z = \phi$

18. State and prove uniform boundless principle.

19. State and prove open mapping theorem.

20. State and prove Unique Hahn-Banach Extension theorem.

S–7521

Sub. Code
22MMA4C2

M.Sc. DEGREE EXAMINATION, APRIL 2025

Fourth Semester

Mathematics

TOPOLOGY–II

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Is the space $\mathbb{R}^n$ locally compact? Justify your answer.

2. Write down the countable intersection property.

3. Define completely regular space with an example.

4. What is the condition for the two compactification to be equivalent?

5. Define Countably locally finite.

6. What is meant by locally discrete?

7. When will you say that the metric space is said to be complete? Give an example.

8. Define an equicontinuous.

9. Define the compact – open topology.

10. Is the space Q of rationals Baire spacve? Justify your answer.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) Let X be a Hansdorff space. Prove that X is locally compact if and only if given x in X, and given a neighborhood U of x, there is neighborhood V of x such that $\overline{V}$ is compact and $\overline{V} \subset U$.

Or

(b) Let X be a set and $\mathcal{D}$ be a collection of subsets of X that is maximal with respect to the finite intersection property. Prove that any finite intersection of elements of $\mathcal{D}$ is an element of $\mathcal{D}$.

12. (a) Prove that a subspace of a completely regular space is completely regular.

Or

(b) If X is completely regular and noncompact, then prove that $\beta(x)$ is not metrizable.

13. (a) Let $\mathcal{A}$ be a locally finite collection of subsets of X. Prove the following:

(i) Any subcollection of $\mathcal{A}$ is locally finite

(ii) The collection $\mathfrak{B} = \{\overline{A}\}_{A \in A}$ of the closures of the elements of $\mathcal{A}$ is locally finite.

(iii) $\overline{\underset{A \in \mathcal{A}}{U A}} = \underset{A \in \mathcal{A}}{U \overline{A}}$

Or

(b) Let X be normal and let A be closed G_S set in X. Prove that there is a continuous function $f : X \to [0,1]$ such that $f(x) = 0$ for $x \in A$ and $f(x) > 0$ for $x \notin A$.

14. (a) If the space Y is complete in the metric d, then prove that the space Y^J is complete in the uniform metric $\overline{\rho}$ corresponding to d.

Or

(b) Let X be a compactly generated space and let (y, d) be a metric space. Prove that $\mathcal{C}(X, Y)$ is closed in Y^X in the topology of compact convergence.

15. (a) Let X be locally compact Hausdorff and let $\mathcal{C}\ (X,Y)$ have the compact – open topology. Prove that the map $\mathcal{C}\ XX\ \mathcal{C}\ (x,y) \to Y$, defined by the equation $\mathcal{C}\ (x, f) = f(x)$ is continuous.

Or

(b) Show that every locally compact Hausdorff space is Baire space.

Part **C** (3 × 10 = 30)

Answer any **three** questions.

16. Prove that an arbitary product of compact spaces is compact in the product topology.

17. Let X be a space. Suppose that $h : x \to z$ is an imbedding of X in the compact Hausdorff space Z. Prove that there exists a corresponding compactification Y of X it has the property that there is an imbedding $H : Y \to Z$ that equals H on X. The compactification Y is uniquely determined up to equivalence.

18. Let X be a metrizable space. If $\mathcal{A}$ is an open covering of X, then prove that there is an open covering ε of X refining $\mathcal{A}$ that is countably locally finite.

19. Show that a metric space (X,d) is compact if and only if it is complete and totally bounded.

20. State and prove Baire Category theorem.

S–7522

Sub. Code
22MMA4C3

M.Sc. DEGREE EXAMINATION, APRIL 2025

Fourth Semester

Mathematics

OPERATIONS RESEARCH

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define a network with an exmaple.

2. What is meant by enumeration of cuts?

3. Define time float.

4. Write down the Red-Flagging rule.

5. What is meant by shortage cost?

6. Define effective lead time.

7. Define queue discipline.

8. Determine the average arrival rate or hour λ of the one arrival occurs every 10 minutes.

9. Write down the little's formula.

10. What is meant by busy servers?

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) Midwest TV cable company is in the process of providing cable service to five new housing development areas. Figure depicts possible TV linkages among the five areas. The cable miles are shown on each arc. Determine the most economical cable network.

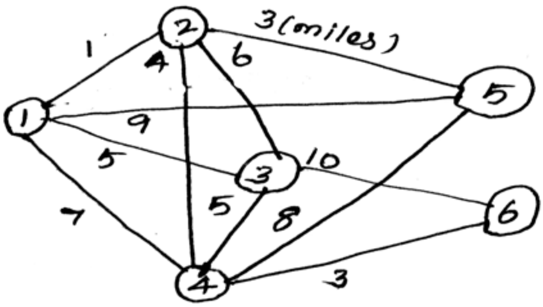
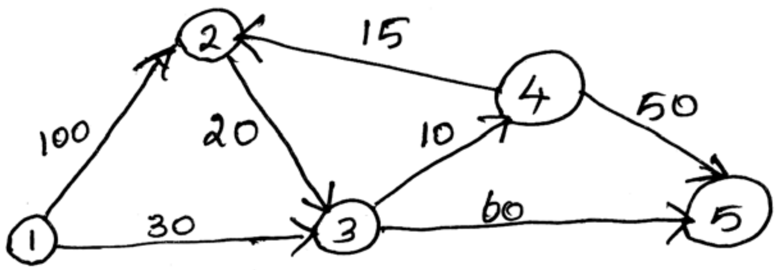


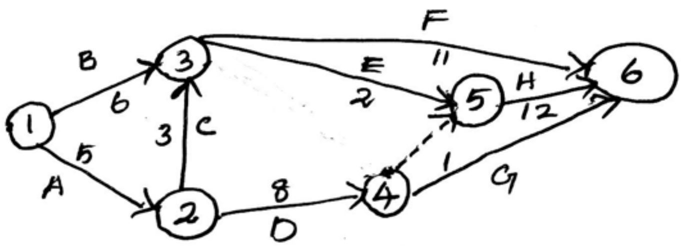
Figure: cable connections for Midwest TV company

Or

(b) The network is figure gives the permissible routes and their lengths in miles between city 1 (node 1) and four other cities (node 2 to 5). Determine the shortest routes between city 1 and each of the remaining four cities.



12. (a) Determine the critical path for the project network in figure. All the durations are in days.



Or

(b) The footings of a building can be completed in 4 consecutive sections. The activities for each section include

(i) Digging

(ii) Placing steel, and

(iii) Pouring concrete.

The digging of one section cannot start unit that of the preceding section has been completed. The same restriction applies to pouring concrete. Develop the project network.

13. (a) Explain multi-item EOQ with storage limitation.

Or

(b) An item sells for \$25 a unit, but a 10% discount is offered for lots of 150 units or more. A company uses this item at the rate of 20 units per day. The setup cost for ordering a lot is \$50, and the holding cost per unit per days is \$30. The lead time is 12 days. Should the company take advantages of the discount?

14. (a) Explain your understanding of the relationship between the arrival rate λ and the average interarrival time what are the units describing each variable?

Or

(b) Derive pure death model.

15. (a) Explain (M/M/1):(GD/N/∞) queuing model.

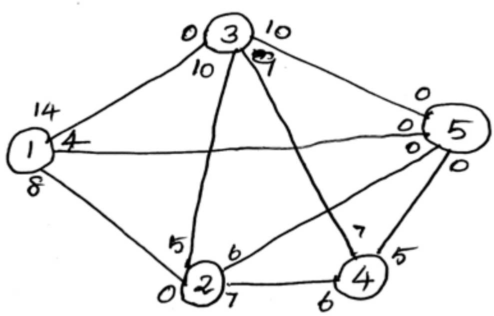
Or

(b) An investor invests \$100 a month on average is one type of stock market security. Because the investor must wait for a good "buy" opportunity, the actual time of purchase is totally random. The investor usually keeps the securities for about 3 years on the average but will sell them at random times when a "sell" opportunity presents itself. Although the investor is generally recognized as a shrewd stock market player, past experience indicates that about 25% of the securities decline at about 20% of a year. The remaining 75% appreciate at the rate of about 12% of a year. Estimate the investor's (long-run) average equity in the stock market.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Determine the maximum flow and the optimum flow in each arc for the network in figure.



17. A project has the following activities and other characteristics.

Activity	i – j	(a, m, d)	Activity	i – j	(a, m, b)
A	1 – 2	(3, 5, 7)	E	3 – 5	(1, 2, 3)
B	1 – 3	(4, 6, 8)	F	3 – 6	(9, 11, 13)
C	2 – 3	(1, 3, 5)	G	4 – 6	(1, 1, 1)
D	2 – 4	(5, 8, 11)	H	5 – 6	(10, 12, 14)

Determine the probabilities that the different nodes of the project will be realized without delay.

18. Neon lights on the U of A campus are replaced at the rate of 100 units per day. The physical plant orders the neon lights periodically. It costs \$100 to initiate a purchase order. A neon light kept in storage is estimated to cost about \$0.02 per day. The load time between placing and receiving an order is 12 days. Determine the optimal inventory policy for ordering the neon lights.

19. Babies are born in a sparsely populated state at the rate of one birth every 12 minutes. The time between births follows an exponential distribution. Find the following :

(a) The average number of births per year.

(b) The probability that no births will occur in any one day.

(c) The probability of issuing 50 birth certificates in 3 hours given that 40 certificates were issued during the first 2 hours of the 3-hours period.

20. Visitor's parking at Ozark college is limited to 5 spaces only. Car making use of this space arrive according to a poission distribution at the rate of 6 cars per hour. parking time is exponentially distributed with a mean of 30 minutes. visitors who cannot find an empty space on arrival may temporarily wait inside the lot until a parked

cat leaves. The temporary space can hold only three cars. Other cars that cannot park or find a temporary waiting space must go elsewhere determine the following

(a) The probability, Pn, of n cars in the system.

(b) The effective arrival rate for cars that actually use the lot.

(c) The average number of cars in the lot.

(d) The average time a car waits for a parking space inside the lot.

(e) The average number of occupied parking spaces.

(f) The average utilization of the parking lot.
